

Common Features of Particle Multiplicities in Heavy Ion Collisions.

*Università di Firenze and INFN Sezione di Firenze,
Largo E. Fermi 2, I-50125, Florence, Italy*

*Department of Physics, University of Cape Town,
Rondebosch 7701, Cape Town, South Africa*

*Department of Physical Sciences, University of Oulu,
FIN-90571 Oulu, Finland*

*Institute of Theoretical Physics, University of Wrocław,
PL-50204 Wrocław, Poland.*

Abstract

Results of a systematic study of fully integrated particle multiplicities in central Au–Au and Pb–Pb collisions at beam momenta of 1.7A GeV, 11.6A GeV (Au–Au) and 158A GeV (Pb–Pb) using a statistical-thermal model are presented. The close similarity of the colliding systems makes it possible to study heavy ion collisions under definite initial conditions over a range of centre-of-mass energies covering more than one order of magnitude. We conclude that a thermal model description of particle multiplicities, with additional strangeness suppression, is possible for each energy. The degree of chemical equilibrium of strange particles and the relative production of strange quarks with respect to u and d quarks are higher than in e^+e^- , pp and $p\bar{p}$ collisions at comparable and even at lower energies. The average energy per hadron in the comoving frame is always close to 1 GeV per hadron despite the fact that the energy varies more than 10-fold.

1 Introduction

It is becoming more and more clear that results from relativistic heavy ion collisions at many different energies[1] show striking common traits. Statistical-thermal models are able to reproduce particle multiplicities in a satisfactory manner by using a very small number of parameters: temperature, volume, baryon chemical potential and a possible strange-quark suppression parameter, γ_s [2]. We report here the results[3] of an analysis of data from collisions at several different energies, with emphasis on the similarity of the colliding system. We have focussed our attention on central Au–Au collisions at beam momenta of 1.7A GeV (SIS) [4], 11.6A GeV (AGS) [5] and on central Pb–Pb collisions at 158A GeV (SPS) beam momentum [6]. As far as the choice of

data (and, consequently, colliding system) is concerned, our leading rule is the availability of full phase space integrated multiplicity measurements because a pure statistical-thermal model analysis of particle yields, without any consideration of dynamical effects, *may* apply only in this case [7]. Such data, however, exist only in a few cases and whenever legitimate we have extrapolated spectra measured in a limited rapidity window to full phase space. The use of extrapolations is more correct than using data over limited intervals of rapidity, especially in the framework of a purely statistical-thermal analysis without a dynamical model. Moreover, the usually employed requirement of zero strangeness ($S = 0$) demands fully integrated multiplicities because strangeness does not need to vanish in a limited region of phase space.

In order to assess the consistency of the results obtained, we have performed the statistical-thermal model analysis by using two completely independent numerical algorithms whose outcomes turned out to be in close agreement throughout. Similar analyses have been recently made by other authors (see e.g. [9, 10]); however, both the model and the used data set differ in several important details, such as the assumption of full or partial equilibrium for some quark flavours, the number of included resonances, the treatment of resonance widths, inclusion or not of excluded volume corrections, treatment of flow, corrections due to limited rapidity windows etc. Because of these differences it is difficult to trace the origin of discrepancies between different results. We hope that the present analysis, covering a wide range of beam energies using a consistent treatment, will make it easier to appreciate the energy dependence of the various parameters such as temperature and chemical potential.

2 Data set and model description

As emphasized in the introduction, in the present analysis we use the most recent available data, concentrating on fully integrated particle yields and discarding data that have been obtained in limited kinematic windows. We have derived integrated multiplicities of π^+ , Λ and proton in Au–Au collisions at AGS by extrapolating published rapidity distributions [12, 13, 14] with constrained mid-rapidity value ($y_{\text{NN}}=1.6$). For proton and Λ we have fitted the data to Gaussian distributions, whilst for π^+ we have used a symmetric flat distribution at midrapidity with Gaussian-shaped wings on either side; the point at which the Gaussian wing and the plateau connect is a free parameter of the fit. The fits yielded very good χ^2 's/dof: 0.27, 1.24 and 1.00 for π^+ , proton and Λ respectively. The integrated multiplicities have

been taken as the area under the fitted distribution between the minimal y_{\min} and maximal y_{\max} values of rapidities for the reactions $NN \rightarrow \pi NN$, $NN \rightarrow \Lambda K$ for pions and Λ 's respectively; the difference between these areas and the total area has been taken as an additional systematic error. The area between y_{\min} and y_{\max} amounts to practically 100% of the total area for pions and about 95% for Λ 's.

We have not included data on deuteron production because of the possible inclusion of fragments in the measured yields. This is particularly dangerous at low (SIS) energies where inclusion or not of deuterons modifies thermodynamic quantities like ϵ/n [15].

The data analysis has been performed within an ideal hadron gas grand-canonical framework supplemented with strange quark fugacity γ_s . In this approach, the overall average multiplicities of hadrons and hadronic resonances are determined by an integral over a statistical distribution:

$$\langle n_i \rangle = (2J_i + 1) \frac{V}{(2\pi)^3} \int d^3p \frac{1}{\gamma_s^{-s_i} \exp[(E_i - \boldsymbol{\mu} \cdot \mathbf{q}_i)/T] \pm 1} \quad (1)$$

where \mathbf{q}_i is a three-dimensional vector with electric charge, baryon number and strangeness of hadron i as components; $\boldsymbol{\mu}$ the vector of relevant chemical potentials; J_i the spin of hadron i and s_i the number of valence strange quarks in it; the $+$ sign in the denominator is relevant for fermions, the $-$ for bosons. This formula holds in case of many different statistical-thermal systems (i.e. clusters or fireballs) having common temperature and γ_s but different arbitrary momenta, provided that the probability of realizing a given distribution of quantum numbers among them follows a statistical rule [8, 16]. In this case V must be understood as the sum of all cluster volumes measured in their own rest frame. Furthermore, since both volume and participant nucleons may fluctuate on an event by event basis, V and $\boldsymbol{\mu}$ (and maybe T) in Eq. (1) should be considered as average quantities [8]. The overall abundance of a hadron of type i to be compared with experimental data is determined by the sum of Eq. (1) and the contribution from decays of heavier hadrons and resonances:

$$n_i = n_i^{\text{primary}} + \sum_j \text{Br}(j \rightarrow i) n_j \quad (2)$$

where the branching ratios $\text{Br}(j \rightarrow i)$ have been taken from the 1998 issue of the Particle Data Table [17].

It must be stressed that the unstable hadrons contributing to the sum in Eq. (2) may differ according to the particular experimental definition. This is a major point in the analysis procedure because quoted experimental multiplicities may or may not include contributions from weak decays of hyperons

and K_S^0 . We have included all weak decay products in our computed multiplicities except in Pb–Pb collisions on the basis of relevant statements in ref. [18] and about antiproton production in refs. [11, 19]. It must be noted that switching this assumption in Au–Au at SIS and AGS does not affect significantly the resulting fit parameters.

The overall multiplicities of hadrons depend on several unknown parameters (see Eq. (1)) which are determined by a fit to the data. The free parameters in the fit are T , V , γ_s and μ_B (the baryon chemical potential) whereas μ_S and μ_Q , i.e. the strangeness and electric chemical potentials, are determined by using the constraint of overall vanishing strangeness and forcing the ratio between net electric charge and net baryon number Q/B to be equal to the ratio between participant protons and nucleons. The latter is assumed to be Z/A of the colliding nucleus in Au–Au and Pb–Pb.

For SIS Au–Au data we have required the exact conservation of strangeness instead of using a strangeness chemical potential. This gives rise to slightly more complex calculations which are necessary owing to the very small strange particle production (Au–Au). The difference between these strangeness-canonical and pure grand-canonical calculations of multiplicities of K and Λ for the final set of thermal parameters (see Table 1) turns out to be as large as a factor 15 in Au–Au at 1.7A GeV.

Owing to few available data points in SIS Au–Au collisions, we have not fitted the volume V nor the γ_s therein. The volume has been assumed to be $4\pi r^3/3$ where $r = 7$ fm (approximately the radius of a Au nucleus) while γ_s has been set to 1, the expected value for a completely equilibrated hadron gas. Since we have performed a strangeness-canonical calculation here, the yield ratios involving strange particle are not independent of the chosen volume value as in the grand-canonical framework. Thus, in this particular case, V is meant to be the volume within which strangeness is conserved (i.e. vanishing) and not the global volume defining overall particle multiplicities as in Eq. (1). Also, in order to test the dependence of this assumption on our results, we have repeated the fit by varying V by a factor 2 and 0.5 in turn.

A major problem in Eq. (2) is where to stop the summation over hadronic states. Indeed, as mass increases, our knowledge of the hadronic spectrum becomes less accurate; starting from ≈ 1.7 GeV many states are possibly missing, masses and widths are not well determined and so are the branching ratios. For this reason, it is unavoidable that a cut-off on hadronic states be introduced in Eq. (2). If the calculations are sensitive to the value of this cut-off, then the reliability of results is questionable. We have performed all our calculations with two cut-offs, one at around 1.8 GeV (in the analysis algorithm A) and the other one at 2.4 GeV (in the analysis algorithm B). The contribution of missing heavy resonances is expected to be very important

for temperatures ≥ 200 MeV making thermal models inherently unreliable above this temperature.

Table 1: Summary of fit results. Free fit parameters are quoted along with resulting minimum χ^2 's and λ_s parameters.

| | Average |
|-------------------------------|---------------------|
| Au–Au 1.7A GeV | |
| T (MeV) | 49.6 ± 2.5 |
| μ_B (MeV) | 813 ± 23 |
| γ_s | 1 (fixed) |
| V (fm ³) | 1437 (fixed) |
| λ_s | 0.0054 ± 0.0035 |
| Au–Au 11.6A GeV | |
| T (MeV) | 119.8 ± 8.3 |
| μ_B (MeV) | 553.5 ± 16 |
| γ_s | 0.720 ± 0.097 |
| $VT^3 \exp(-0.7\text{GeV}/T)$ | 2.03 ± 0.34 |
| λ_s | 0.43 ± 0.10 |
| Pb–Pb 158A GeV | |
| T (MeV) | 158.1 ± 3.2 |
| μ_B (MeV) | 238 ± 13 |
| γ_s | 0.789 ± 0.052 |
| $VT^3 \exp(-0.7\text{GeV}/T)$ | 21.7 ± 2.6 |
| λ_s | 0.447 ± 0.025 |

3 Results

As mentioned in the introduction, we have performed two analyses (A and B) by using completely independent algorithms.

In the analysis A all light-flavoured resonances up to 1.8 GeV have been included. The production of neutral hadrons with a fraction f of $s\bar{s}$ content has been suppressed by a factor $(1 - f) + f\gamma_s^2$. In the analysis B the mass cut-off has been pushed to 2.4 GeV and neutral hadrons with a fraction f of $s\bar{s}$ content have been suppressed by a factor γ_s^{2f} . Both algorithms use masses, widths and branching ratios of hadrons taken from the 1998 issue of Particle Data Table [17]. However, it must be noted that differences between the two analyses exist in dealing with poorly known heavy resonance parameters,

such as assumed central values of mass and width, where the Particle Data Table itself gives only a rough estimate. Moreover, the two analyses differ by the treatment of mass windows within which the relativistic Breit-Wigner distribution is integrated.

A summary of the final results is shown in Fig. 1. For each analysis an estimate of systematic errors on fit parameters have been obtained by repeating the fit

- assuming vanishing widths for all resonances
- varying the mass cut-off to 1.7 in analysis A and to 1.8 in analysis B
- for Au–Au at 1.7A GeV, the volume V has been varied to $V/2$ and to $2V$ (see discussion in Sect. 2)

The differences between new fitted parameters and main parameters have been conservatively taken as uncorrelated systematic errors to be added in quadrature for each variation (see Table 1). The effect of errors on masses, widths and branching ratios of inserted hadrons has been studied in analysis A according to the procedure described in ref. [8] and found to be negligible. Finally, the results of the two analyses have been averaged according to a method suggested in ref. [22], well suited for strongly correlated measurements.

4 Discussion and conclusions

From the results obtained, an indication emerges that a statistical-thermal description of multiplicities in a wide range of heavy ion collisions is indeed possible to a satisfactory degree of accuracy, for beam momenta ranging from 1.7A GeV to 158A GeV per nucleon. Furthermore, the fitted parameters show a remarkably smooth and consistent dependence as a function of centre-of-mass energy.

The temperature varies considerably between the lowest and the highest beam energy, namely, between 50 MeV at SIS and 160 MeV at SPS. Similarly, the baryon chemical potential changes appreciably, decreasing from about 820 MeV at SIS to about 240 MeV at SPS. However, since the changes in temperature and chemical potential are opposite, the resulting energy per particle shows little variation and remains practically constant at about 1 GeV per particle; this is shown in Fig. 2.

The supplementary γ_s factor, measuring the deviation from a completely equilibrated hadron gas, is around 0.7 – 0.8 at all energies where it has been considered a free fit parameter. At the presently found level of accuracy, a

fully equilibrated hadron gas (i.e. $\gamma_s = 1$) cannot be ruled out in all examined collisions except in Pb–Pb, where γ_s deviates from 1 by more than 4σ . This result does not agree with a recent similar analysis of Pb–Pb data [9] imposing a full strangeness equilibrium. The main reason of this discrepancy is to be found in the different data set used; whilst in ref. [9] measurements in different limited rapidity intervals have been collected, we have used only particle yields extrapolated to full phase space. The temperature values that we have found essentially agree with previous analyses in Au–Au collisions [20] and estimates 11.7 A GeV [23].

The T value in Pb–Pb is strongly affected by high mass particle measurements, such as ϕ and Ξ . A recent significant lowering of the Ξ yield measured by NA49 [21] with respect to a previous measurement [24] results in a decrease of estimated temperature value from about 180 MeV to the actual 160 MeV.

Forthcoming lower energy Pb–Pb and high energy Au–Au data at RHIC should allow to clarify the behaviour of strangeness production in heavy ion collision.

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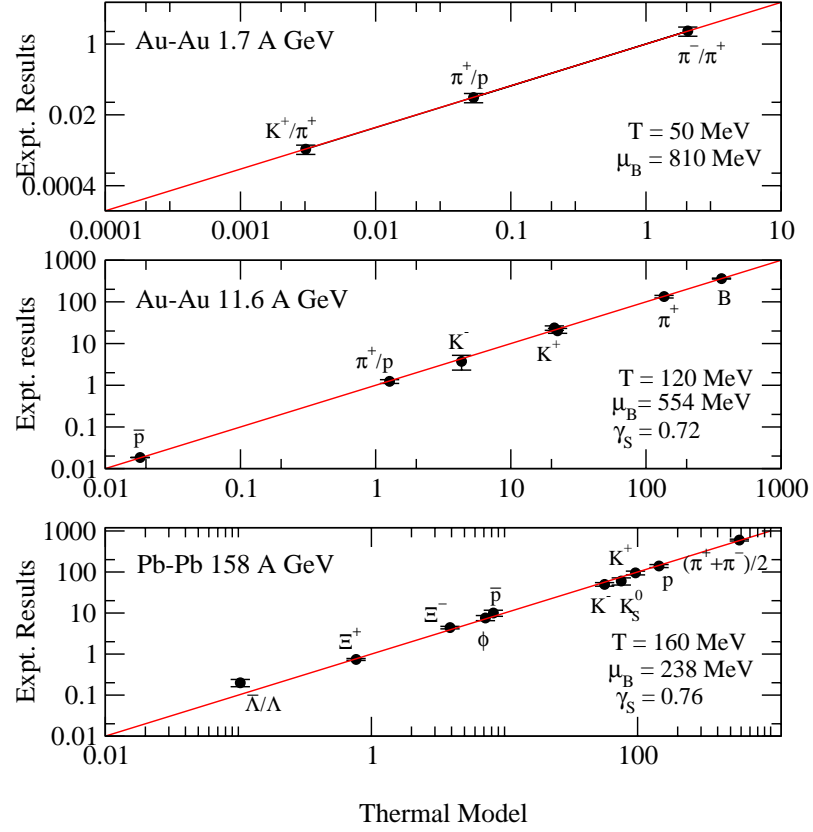


Figure 1: Comparison between particle multiplicities fitted using the thermal model and experimental results.

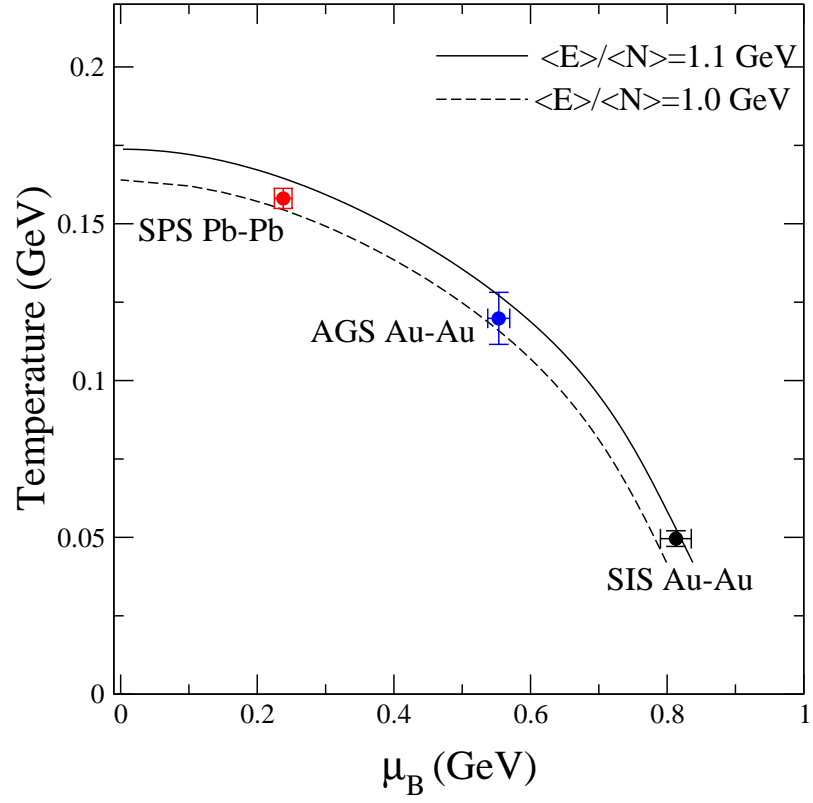


Figure 2: Fitted temperatures and baryon-chemical potentials plotted along with curves of constant energy per hadron.